Combinatorics

Instructions: All questions carry equal marks.

1. Define *mutually orthogonal Latin squares*. Prove that there can be at most $n-1$ mutually orthogonal Latin squares of order n and show that this bound is attained when n is a prime power.

2. (*Fisher's inequality*) Prove that for a $2-(v,k,\lambda)$ design with b blocks and $v > k$, we must have $b \geq v$.

3. Define an *independent set* and a *basis* of a flat in a combinatorial geometry. Prove that in a geometry, basis of a flat exists and that any two bases of a flat have the same cardinality.

4. Define a *modular* geometry. Prove that a finite combinatorial geometry of rank n is modular if and only if any rank 2 flat (a line) and rank $n-1$ flat (a hyperplane) meet nontrivially.

5. Classify all finite modular geometries of rank 3.